Int. J. Heat Mass Transfer. Vol. 14, pp. 377–385. Pergamon Press 1971. Printed in Great Britain

STABILITY OF LIQUID FLOW DOWN A HEATED INCLINED PLANE

S. GEORGE BANKOFF*
Consultant, Reactor Safety Div. Argonne National Laboratory, Argonne, Illinois, U.S.A.

(*Received* 8 *April* 1970 *and in revised form* 15 *May* 1970)

Abstract—The Yih–Benjamin analysis of the stability to small surface perturbations of a thin liquid film flowing down an inclined wall is extended to take into account the effects of evaporation or condensation. Evaporation destabilizes the film. With vertical walls the flow is thus unstable at every Reynolds number under heating, but exhibits a critical Reynolds number under cooling. Above a critical heat flux the estimated time to develop thin spots decreases as the fourth power of the evaporation rate.

NOMENCLATURE

A,	dimensionless constant;
$c = c_r + ic_i$,	complex wave velocity;
d,	mean film thickness;
F	Froude number;
f,	pressure amplitude;
g,	acceleration due to gravity;
k,	liquid thermal conductivity;
L_{b0}.	length for formation of a thin spot;
P,	steady dimensionless pressure;
p_1,	dimensionless pressure;
R,	Reynolds number;
R_c,	critical Reynolds number;
q_s,	wall heat flux;
S,	dimensionless surface tension, equation (18);
T,	temperature;
t,	time;
t_g,	estimated time for film thinning;
U,	normalized steady velocity in X-direction;
$\bar{u}$,	velocity in X-direction;
u_1,	dimensionless velocity in X-direction;
$\bar{v}$,	velocity in Y-direction;
v_1,	dimensionless velocity in Y-direction;
X,	distance in direction of mean flow;
x,	X/d;
Y,	distance from mean surface position;
y,	Y/d.

Greek letters

α,	wave number;
β,	angle of inclination with horizontal;
β_1,	equivalent angle of inclination, equation (46);
γ,	ρ_g/ρ;
δ,	dimensionless heat flux term, equation (43);
δ_1,	dimensionless parameter, equation (51);
Δ,	Laplacian operator;
ε,	dimensionless initial surface wave amplitude;
η,	surface displacement [dimensionless];
θ,	dimensionless temperature:
λ_g,	heat of vaporization;
ν,	kinematic viscosity;

* Chemical Engineering department, Northwestern University, Evanston, Illinois.

ρ,	liquid density;
ρ_g,	vapor density;
σ,	surface tension;
τ,	dimensionless time;
τ_g,	growth time;
φ,	stream function perturbation amplitude;
ψ,	stream function;
ω,	dimensionless parameter, equation (24).

Subscripts

a,	spatial average;
c,	critical;
e,	evaporative;
s,	surface;
w,	wall;
0,	absence of heating.

Superscripts

$'$,	perturbation quantity;
$*$,	most dangerous wave (greatest growth rate).

1. INTRODUCTION

THE STABILITY of a thin liquid film adhering to a heated wall is of practical importance in several applications. One example can be drawn from the field of sodium-cooled fast reactor safety analysis, where, as the result of a hypothetical accident involving either loss of coolant flow or a power excursion, rapid formation of a vapor slug takes place, which expels liquid from the coolant channel. A thin film of residual liquid will be left adhering, at least for short times, to the wall [1]. Calculations [2, 3] have shown that the stability of this film crucially affects the rate of expulsion. In particular, the time for breakthrough of vapor out of the core section may be an order of magnitude greater when the liquid film is absent than when it remains on the wall throughout the expulsion.

The only sodium film measurements presently available are those of Spiller *et al.* [1], who subjected a stagnant column of liquid to transient heating, and observed film thicknesses of 0·07–0·25 mm. A simple calculation shows that films of this thickness attain their steady-state drainage velocity profiles in times of the order of 10^{-2} s, whereas the total expulsion occurs in times of the order of 0·1 s. Clearly, then, one may neglect the initial period during which the velocity profile is being developed, and concentrate instead upon the stability of the sodium film as it flows steadily down the heated wall. The purely fluid-mechanical problem, in the absence of heat transfer, is well-known, having been studied first by Yih [4] who gave numerical solutions, and then by Benjamin [5]. The latter obtained analytical neutral-stability curves, which established that free-surface flow down a vertical plane is unstable for all Reynolds numbers.

In a later paper Yih [6] provided a perturbation procedure which is considerably simpler than Benjamin's power-series expansion, and thereby extended the problem in several significent ways. We follow here the general framework provided by Yih in considering the combined fluid-mechanical and heat transfer problem. Because of the heat flow from the wall, vaporization will occur at the free liquid surface. If the effects of the temperature variation in the liquid on the liquid physical properties are ignored, it is clear that the effect of the heated wall will appear only in the pressure boundary condition at the free surface, rather than in the equations of motion. Here it will have a destabilizing effect, as can be seen from the following simple considerations. The instantaneous vaporization rate will be greater at the troughs, rather than at the crests, of a surface wave, since the film is thinner at these points. The vapour leaving normal to the surface exerts a reactive pressure on the liquid, which is therefore larger at the troughs than at the crests, and hence tends to increase the wave amplitude.

A further extension of the prior analyses considers the rate of growth of the most dangerous wavelength. From this one can estimate the fractional change, due to heating, in the time for the film to develop thin spots. The second stage of film break-up involves the wettability of the wall, as evidenced by the liquid contact angle, and hence requires a different analysis. It is of some interest that Freon 113 is a considerably better simulant of sodium film thinning than water, providing the heat flux is reduced by an order of magnitude.

2. FORMULATION OF THE PROBLEM

Consider a liquid film of mean thickness d, draining steadily down a heated plane inclined at an angle β with the horizontal. Let X be the distance in the direction of mean flow, and Y the distance from the mean surface position, with $\bar{u}$ and $\bar{v}$ the corresponding velocity components. The steady velocity profile (with the assumption of zero surface shear stress) is thus given by

$$U(y) = \tfrac{3}{2}(1 - y^2) \tag{1}$$

where $y = \gamma/d$ $U \triangleq \bar{u}/\bar{u}_a$ is the normalized steady velocity. The normalization is performed with respect to the average velocity parallel to the wall. Letting ν be the kinematic viscosity, it is readily shown that

$$\bar{u}_a = \frac{gd^2 \sin\beta}{3\nu}. \tag{2}$$

Defining the Reynolds number and Froude number by

$$R = \frac{\bar{u}_a d}{\nu}; \qquad F = \frac{\bar{u}_a}{(gd)^{\frac{1}{2}}} \tag{3}$$

it is clear that

$$3F^2 = R \sin\beta, \tag{4}$$

so that only the Reynolds number can be chosen independently.

Consider a small surface wave, of amplitude ηd, where η is the normalized displacement of the surface from its mean position. The velocities, distances, pressures and time can similarly be made dimensionless be defining

$$(u_1, v_1) = \frac{(u, v)}{\bar{u}_a}; \qquad (x, y) = \frac{(X, Y)}{d}$$

$$p_1 = \frac{p}{\rho \bar{u}_a^2}; \qquad \tau = \frac{t\bar{u}_a}{d}. \tag{5}$$

The equations of motion and of continuity are then:

$$\frac{\partial u_1}{\partial \tau} + u_1\frac{\partial u_1}{\partial x} + v_1\frac{\partial u_1}{\partial y} = -\frac{\partial p_1}{\partial x} + \frac{\sin\beta}{F^2} + \frac{1}{R}\Delta u_1 \tag{6}$$

$$\frac{\partial v_1}{\partial \tau} + u_1\frac{\partial v_1}{\partial x} + v_1\frac{\partial v_1}{\partial y} = -\frac{\partial p_1}{\partial y} + \frac{\cos\beta}{F^2} + \frac{1}{R}\Delta v_1 \tag{7}$$

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0. \tag{8}$$

Let the steady (dimensionless) pressure be P everywhere. The presence of the surface wave implies a perturbation of the steady flow field such that

$$u_1 = U + u'; \qquad v_1 = v'; \qquad p = P + p'. \tag{9}$$

Defining the perturbation stream functions by

$$u' = \psi_y; \qquad v' = \psi_x \tag{10}$$

and substituting equations (9) and (10) into equations (6) and (7) [equation (8) is automatically satisfied], one obtains, upon neglecting the products of perturbation quantities:

$$\psi_{y\tau} + U\psi_{xy} - U_y\psi_x = -p'_x + \frac{1}{R}\Delta\psi_y \tag{11}$$

$$\psi_{x\tau} + U\psi_{xx} = p'_y + \frac{1}{R}\Delta\psi_x. \tag{12}$$

At the bottom of the film ($y = 1$)

$$u' = \psi_y = 0; \qquad v' = -\psi_x = 0. \tag{13}$$

At the free surface, the vanishing of the shear stress implies that

$$\frac{\partial v_1}{\partial x} + \frac{\partial u_1}{\partial y} = 0. \tag{14}$$

Let V_e be the velocity component of the vapor leaving the surface due to evaporation, and σ be the surface tension. The pressure condition at the free surface then requires that

$$\left(-p_1 + \gamma v_e^2 + \frac{2}{R}\frac{\partial v_1}{\partial y}\right)\rho \bar{u}_a^2 + \sigma \frac{\partial^2(\eta d)}{\partial x^2} = 0 \tag{15}$$

where

$$\gamma = \frac{\rho_g}{\rho}; \qquad v_e = \frac{V_e}{\bar{u}_a}. \tag{16}$$

γ, which is the ratio of the vapor to the liquid density, is normally quite small, so that the reactive pressure term due to evaporation might be considered to be small. However, at sufficiently high heat fluxes, it will be shown to be of decisive importance. In dimensionless form, equation (15) may be written

$$-p_1 + \gamma v_e^2 + \frac{2}{R}\frac{\partial v_1}{\partial y} + S\frac{\partial^2 \eta}{\partial x^2} = 0 \tag{17}$$

where

$$S = \frac{\sigma}{\rho d \bar{u}_a^2}. \tag{18}$$

Equations (14) and (17) are strictly correct only at the free surface ($y = \eta$); and in order to obtain boundary conditions at the fixed elevation, $y = 0$, we take advantage of the fact that quadratic terms in the Taylor series expansions can be neglected, by virtue of the assumption of $\eta \ll 1$. Equations (14) and (15) then become

$$\frac{d^2 U}{dy^2}\eta + \psi_{yy} - \psi_{xx} = 0 \tag{19}$$

and

$$-P - P_y \eta - p' + \gamma \bar{v}_e^2 + 2\gamma \bar{v}_e v_e' - \frac{2}{R}\psi_{xy} + S\eta_{xx} = 0 \tag{20}$$

where $v_e = \bar{v}_e + v_e'$, and the steady pressure and pressure gradient at the surface of the undisturbed flow are determined by the evaporative momentum flux and gravity forces, respectively:

$$P(0) = \gamma \bar{v}_e^2; \qquad P_y(0) = \frac{\cos\beta}{F^2}. \tag{21}$$

Inserting equation (21) into equation (20), we obtain the perturbation pressure condition at $y = 0$:

$$\frac{\cos\beta}{F^2}\eta + p' - 2\gamma \bar{v}_e v_e' + \frac{2}{R}\psi_{xy} - S\eta_{xx} = 0. \tag{22}$$

We turn now to the heat flow problem. Letting $\theta = \frac{T - T_s}{T_w - T_s}$, where T is the temperature of the liquid, and T_w and T_s are the (constant) temperatures of the wall and surface, respectively, the instantaneous evaporative flux can be related to the surface temperature gradient by

$$\theta_y|_{y=\eta} = -\omega(\bar{v}_e + v_e') \tag{23}$$

where

$$\omega = \frac{\lambda_g \rho_g d \bar{u}_a}{k(T_w - T_s)} \tag{24}$$

and λ_g, k are the heat of vaporization and liquid thermal conductivity, respectively. For the steady flow the surface temperature gradient is given by

$$\bar{\theta}_y|_{y=0} = -\omega \bar{v}_e \tag{25}$$

which implies, together with equation (23), that

$$v_e' = -\theta_{yy}\Big|_{y=0}\frac{\eta}{\omega} \simeq -\frac{1}{\omega}\bar{\theta}_y\Big|_{y=0}^{y=\eta}. \tag{26}$$

In fact, the temperature field will be disturbed by the flow perturbation, but, as can be seen from equation (26), this constitutes a second-order correction on the evaporative flux, and may be neglected. The steady surface temperature gradient will be inversely proportional to the film thickness, so that

$$\frac{\bar{\theta}_y|_{y=\eta}}{\bar{\theta}_y|_{y=0}} \simeq \frac{1}{1-\eta}. \tag{27}$$

Combining equations (25)–(27), one obtains

$$v_e' = \bar{v}_e\left(\frac{1}{1-\eta} - 1\right) \simeq \bar{v}_e\eta. \tag{28}$$

Equation (22) thus becomes

$$\frac{\cos\beta}{F^2}\eta + p' - \frac{2\gamma}{\omega^2}\bar{\theta}_y^2\eta + \frac{2}{R}\psi_{xy} - Sn_{xx} = 0. \tag{29}$$

Now, any Fourier component of the disturbance can be written in the form

$$\psi = \varphi(y)\exp\left[i\alpha(x - c\tau)\right] \tag{30}$$

$$p' = f(y)\exp\left[i\alpha(x - c\tau)\right] \tag{31}$$

$$\eta = \mu\exp\left[i\alpha(x - c\tau)\right] \tag{32}$$

where

$$c = c_r + ic_i \tag{33}$$

is a complex wave velocity, and

$$\alpha = \frac{2\pi d}{\lambda} \tag{34}$$

is a dimensionless wave number.

The kinematic condition at the free surface

$$-\psi_x = \eta_\tau + U\eta_x \tag{35}$$

leads to

$$\varphi(0) = \mu[c - U(0)] \tag{36}$$

where, from equation (1), $U(0) = 3/2$. Letting $c' = c - \frac{3}{2}$, we have

$$\mu(0) = \frac{\varphi(0)}{c'}. \tag{37}$$

The equations of motion (11) and (12), upon substituting (30)–(32), and eliminating the pressure term, yield the well-known Orr–Sommerfeld equation:

$$\varphi'''' - 2\alpha^2\varphi'' + \alpha^4\varphi = i\alpha R[(U - c)(\varphi'' - \alpha^2\varphi) - U''\varphi] \tag{38}$$

with boundary conditions from (13) and (19)

(i) $\varphi'(1) = 0$ (39)

(ii) $\varphi(1) = 0$ (40)

(iii) $\chi(0) = \varphi''(0) + \left(\alpha^2 - \frac{3}{c'}\right)\varphi(0) = 0$ (41)

since $U(0) = \frac{3}{2}$; $U_y(0) = 0$ and $U_{yy}(0) = -3$. The fourth b.c. is obtained from (29), noting that $\cos\beta/F^2 = 3\cot\beta/R$:

$$\alpha\varphi(0)(3\cot\beta + \alpha^2 SR - 2\delta R) + \alpha(Rc' + 3\alpha i)\varphi'(0) - i\varphi'''(0) = 0 \tag{42}$$

where

$$\delta \triangleq \frac{\gamma\bar{\theta}_y^2}{\omega^2} = \frac{\rho_g}{\rho}\left(\frac{q_s}{\lambda_g\rho_g\bar{u}_a}\right)^2 \tag{43}$$

and q_s is the wall heat flux.* The case $\delta = 0$ corresponds to the Yih problem. There is thus an extra degree of freedom compared to the problem considered by Yih and by Benjamin, since

$$c_i = c_i(R, F, \alpha, \delta) = 0 \tag{44}$$

gives the neutral stability curve in the (R, F, α, δ) hyperplane.

3. THIN FILM APPROXIMATION

For the extremely thin films quoted above [1], it is clear that surface tension will quickly damp out waves whose dimensionless wave number is of order unity. Hence, we need concern ourselves only with long waves, such that $\alpha \ll 1$. Considering α to be a small perturbation

* An examination of the right hand side of equation (43) shows that δ is only very weakly affected by deviations of the liquid surface temperature, T_s, from saturation, due to the evaporating or condensing mass flux. In addition, variations in surface tension with x (Marangoni effects) have been neglected in this analysis. To justify this, suppose that $S_x \neq 0$. Then equation (22) will contain an additional term on the left hand side which is $-S_x\eta_x$.

The linearized non-equilibrium transport equation becomes, at every surface point,

$$A\theta|_{y=\eta} = \bar{\theta}_y|_{y=\eta} \cong \bar{\theta}_y|_{y=0}(1-\eta)^{-1}$$

where A is a dimensionless constant involving the accommodation coefficient, the molecular weight, the saturation temperature and the ambient pressure. This implies that

$$\theta_x|_{y=\eta} \cong A^{-1}\bar{\theta}_y|_{y=0}\,\eta_x$$

and hence that $S_x\eta_x = S_\theta\theta_x|_{y=\eta}\eta_x$ is $O(\alpha^2)$, in view of equation (32). There will thus be (as might be expected) an additional cubic term in equation (47) arising from surface tension effects, which, to the present order of approximation, may also be neglected in determining the critical Reynolds number.

parameter, the zeroth-order approximation of equations (38)–(42) becomes

$$\phi_0'''' = 0$$

$$\left.\begin{aligned} &\text{(i)} \quad \phi_0(1) = 0 \\ &\text{(ii)} \quad \phi_0'(1) = 0 \\ &\text{(iii)} \quad \phi_0''(0) - \frac{3}{c'}\phi_0(0) = 0 \\ &\text{(iv)} \quad \phi_0'''(0) = 0. \end{aligned}\right\} \quad (45)$$

Defining an equivalent angle of inclination, β_1, by

$$3 \cot \beta_1 = 3 \cot \beta - 2\delta R \quad (46)$$

the problem reduces to that considered by Yih. We can therefore employ his result directly:

$$c_{li} = \frac{6\alpha R}{5} - \frac{\alpha}{3}(3 \cot \beta_1 + \alpha^2 SR) \quad (47)$$

where $c_{li} = 0$ is the first-order neutral stability curve. For $\alpha \ll 1$, we can neglect the cubic term in determining the critical Reynolds number R_c:

$$R_c \simeq \frac{\cot \beta}{\frac{6}{5} + \frac{2}{3}\delta}. \quad (48)$$

Above this Reynolds number some disturbance will grow. Hence, for vertical walls ($\beta = \pi/2$), the flow is always unstable. This is not surprising, since as can be seen from equation (48), surface evaporation is always destabilizing. On the other hand, if the direction of heat flow were reversed, so that $\delta < 0$, representing a condensing heat flux, it is interesting to note that one can unconditionally stabilize a draining film, provided $\beta < \pi/2$.

We consider now the most dangerous wavelength, in order to obtain an estimate of the time for thin spots to appear in the film. This is defined as the real root of

$$\frac{d}{d\alpha}(\alpha c_{li}) = 0 \quad (49)$$

which, upon substituting equation (47), gives for the greatest rate of amplification:

$$\alpha^* c_{li}^* = \frac{3(\delta_1 R - \cos \beta)^2}{4SR} \quad (50)$$

where

$$\delta_1 = \frac{6}{5} + \frac{2}{3}\delta. \quad (51)$$

Let

$$\eta^* = \varepsilon \exp\left[i\alpha^*(x - c^*\tau)\right] \quad (52)$$

where $\varepsilon > 0$ is the dimensionless initial surface wave amplitude. An estimate of the growth time for the wave amplitude to be comparable to the film thickness is then obtained by setting $\eta^*(\tau_g) = 1$, whence

$$\tau_g = \frac{-\ln \varepsilon}{\alpha^* c_{1i}^*} = \frac{-4SR.\ln \varepsilon}{3(\delta' R - \cot \beta)^2}. \quad (53)$$

This estimate should be considered to be only a rough guide, since it involves an extrapolation into a range where the neglected quantities are no longer necessarily small.

In physical variables, this becomes for vertical walls:

$$t_g = \frac{\left(-\frac{4}{3}\ln \varepsilon\right)\frac{\sigma \nu}{\rho d^2 \bar{u}_a^3}}{\left[\frac{6}{5} + \frac{2}{3}\frac{\rho_g}{\rho}\left(\frac{q_s}{\lambda_g \rho_g \bar{u}_a}\right)^2\right]^2}. \quad (54)$$

For moderate heat fluxes, the second term in the denominator can be neglected, which corresponds to the zero heat flux case. However, once q_s exceeds a critical value, q_{sc}, given by

$$\frac{q_{sc}}{\lambda_g \rho_g \bar{u}_a} = \left(\frac{9}{5}\frac{\rho}{\rho_g}\right)^{\frac{1}{2}} \quad (55)$$

the growth time falls drastically, and in fact, decreases as the fourth power of the applied heat flux.

Defining now the estimated time for film thinning in the absence of wall heating by t_{g0}, equations (54) and (55) become

$$\frac{t_g}{t_{g0}} = \left[1 + \left(\frac{q_s}{q_{sc}}\right)^2\right]^{-2} \quad (56)$$

where

$$t_{g0} = \frac{25}{27}\left(\frac{\sigma\nu}{\rho d^2 \bar{u}_a^3}\right) \ln \varepsilon \qquad (57)$$

For condensation, rather than evaporation, the sign of the second term on the right-hand side of equation (56) would be reversed, so that the critical condensing heat flux would here correspond to complete stabilization.

4. ILLUSTRATIVE CALCULATIONS

To illustrate the magnitude of these effects, we present some simple illustrative calculations.

Consider a liquid film flowing steadily down a vertical surface at atmospheric pressure, with the free liquid surface being at the saturation temperature as the result of a uniform wall heat flux, q_s. Let $d = 0{\cdot}01$ cm, and $\epsilon = 0{\cdot}01$. The following table, based upon equations (54)–(57), compares the behavior of three different liquids:

Table 1.

Liquid	Growth time, adiabatic walls, t_{g0} (s)	Critical heat flux, q_{sc} (cal/cm^2)	$\bar{u}_a$ (cm/s)	$L_{b0} \triangleq \bar{u}_a t_g{}^0$ (cm)
Freon-113	$6{\cdot}25 \times 10^{-5}$	54·2	14·7	$9{\cdot}2 \times 10^{-4}$
Sodium	$2{\cdot}7 \times 10^{-5}$	530	31·2	$8{\cdot}3 \times 10^{-4}$
Water	5·5	113	6·4	35

Freon-113 is clearly a much better simulant for sodium film breakup than water, provided the heat flux is reduced by a factor of approximately ten. The reason for the large differences in the wave growth time between water and the other fluids can be ascertained from equations (2) and (57), where it is seen that $t_{g0} \sim \nu^4$. It is instructive to compare the calculations of Table 1 for the length for full development of the wave amplitude under adiabatic conditions, L_{b0}, with the recent results of Simon and Hsu [7]. They found a minimum stable thickness of 0·014 cm for an annular film of water on the inside of an unheated glass tube (1·1 cm i.d.), which is in good agreement with the result of Norman and McIntyre [8] for a copper pipe. The test section length in the former study was about 39 cm, so that the agreement with the calculated value of 35 cm is clearly fortuitous, in view of our arbitrary assumption of $\varepsilon = 0{\cdot}01$ and our extrapolation beyond the small-perturbation range. The dependence upon the initial dimensionless surface wave amplitude is, however, logarithmic, so that decreasing ε by a factor of ten increases L_{b0} by only 50 per cent.

One can thus postulate that the film breakup occurs in two stages: (1) an initial stage in which thin spots are produced in the film by growth of an unstable surface wave, (2) a breakup stage in which liquid is displaced from the solid surface. The contact angle, which is a measure of the relative surface energies, is clearly of considerable importance in the second stage, whereas it is irrelevant in the first. The second stage may proceed by means of a minimization of the sum of surface and kinetic energies, as suggested by Hartley and Murgatroyd [9], and may lead to quite asymmetric liquid distribution, as noted by Simon and Hsu [7].

From Table 1 it is seen that thin-spot formation in either Freon-113 or sodium is very rapid. On the other hand, with well-wetted surfaces the film appears to be quite stable, as indicated by the depressurization studies of Grolmes and Fauske [10] with Freon 113 and of Spiller *et al.* [1] with sodium.

5. CONCLUSIONS

It has been shown that evaporation from the surface of a falling liquid film has a destabilizing effect, while condensation has the opposite influence. For every angle of inclination, $\beta < \pi/2$, of a heated wall, there is a corresponding angle of inclination, β_1, given by equation (46), of an unheated wall, for which the stability behavior and amplitude growth rates are exactly equal at all Reynolds numbers, provided that the film thickness is small compared to the wave length. One can estimate thereby, in a very rough way, the minimum time for formation of thin spots in the liquid film, which is certainly a lower bound for the lifetime of the unbroken film. In the second stage a dry spot may or may not be formed, depending upon wettability considerations. This stage is probably quite rapid, although very little evidence is presently at hand.

ACKNOWLEDGEMENTS

Discussions with H. K. Fauske concerning the application of this analysis to fast reactor safety, as well as the thoughtful comments of the reviewers, are appreciated.

REFERENCES

1. K. H. Spiller, G. Grass and D. Perschke, Überhitzung and einzelblasenejektion bei der verdampfung von stagnierenden flussingmetallen, *Atomkernenergie* **12**, 111 (1967).
2. W. D. Ford, H. K. Fauske and S. G. Bankoff, The slug expulsion of Freon-113 by rapid depressurization of a vertical tube. *Int. J. Heat Mass Transfer* **14**, 133–140 (1971).
3. A. W. Cronenberg, III, Slug expulsion of boiling sodium from a reactor coolant channel, M.S. Thesis, Northwestern University, Evanston, Illinois (1969).
4. C.-S. Yih, Stability of parallel laminar flow with a free surface, Proc. 2nd U.S. National Congress Appl. Mech., ASME, 622 (1955).
5. T. Brooke Benjamin, Wave formation in laminar flow down an inclined plane, *J. Fluid Mech.* **2**, 551 (1957).
6. C.-S. Yih, Stability of liquid flow down an inclined plane, *Physics Fluids* **6**, 321 (1963).
7. F. F. Simon and Y. Y. Hsu, Thermocapillary induced breakdown of a falling liquid film, NASA TN D-5642 (1970).
8. W. S. Norman and V. McIntyre, Heat transfer to liquid film on a vertical surface, *Trans. Inst. Chem. Engrs* **38**, 301 (1960).
9. D. E. Hartley and W. Murgatroyd, Criteria for the break-up of thin liquid layers flowing isothermally over solid surfaces, *Int. J. Heat Mass Transfer* **7**, 1003 (1964).
10. M. A. Grolmes and H. K. Fauske, Reactor development program progress report, ANL-7548, p. 118 (1969).

STABILITE DE L'ECOULEMENT D'UN LIQUIDE LE LONG D'UN PLAN INCLINÉ CHAUFFE

Résumé—On étend l'analyse de Yih–Benjamin, concernant la stabilité des perturbations à la surface d'un film liquide mince, s'écoulant le long d'une paroi inclinée pour considérer les effets d'évaporation ou de condensation. L'évaporation déstabilise le film. Avec des parois verticales, l'écoulement est alors instable sous l'échauffement pour tout nombre de Reynolds, mais montre un nombre critique sous le refroidissement. Au-dessus d'un flux critique de chaleur le temps estimé pour développer des taches fines décroît comme la quatrième puissance du taux d'évaporation.

STABILITÄT EINER FLÜSSIGKEITSSTRÖMUNG AN EINER BEHEIZTEN GENEIGTEN PLATTE

Zusammenfassung—Die Yih–Benjamin-Analysis über die Stabilität eines Flüssigkeitsfilms, der an einer geneigten Wand herabfliesst, wird für kleine Störungen der Fläche erweitert, um den Einfluss von Verdampfung und Kondensation zu berücksichtigen. Verdampfung zerstört den Film. An senkrechten Wänden wird demnach bei jeder Reynolds-Zahl die Strömung beim Aufheizen instabil, während bei Kühlung eine kritische Reynolds-Zahl auftritt. Oberhalb einer kritischen Wärmestromdichte nimmt die Zeit für die Entwicklung einer Störstelle mit der vierten Potenz des Verdampfungsverhältnisses ab.

УСТОЙЧИВОСТЬ ПОТОКА ЖИДКОСТИ НА НАГРЕТОЙ НАКЛОННОЙ ПЛОСКОСТИ

Аннотация—Метод Йи-Бенджамина для анализа устойчивости при малых возмущениях на поверхности тонкой пленки жидкости, стекающей по наклонной стенке, обобщается для учета влияния испарения или конденсации. Испарение нарушает устойчивость пленки. Таким образом, в случае вертикальных стенок при нагревании поток является нестационарным при любых числах Рейнольдса, а при охлаждении появляется критическое число Рейнольдса. Выше значения критического теплового потока рассчитанное время, необходимое для появления тонких пятен, уменьшается пропорционально четвертой степени скорости испарения.